Summary RFT III

Antennas

wave detachment

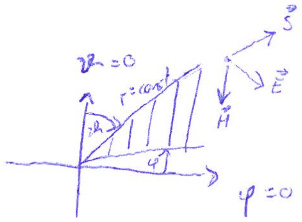
$$\underline{U} = \underline{U}_h \cdot e^{-j\frac{2\pi}{\lambda}z} \qquad \underline{U}_h = |U_h| e^{j\dots}$$

$$u = Re\{\underline{U} \cdot e^{j\omega t}\} = |U_h| \cos(\omega t - \frac{2\pi}{\lambda} z + \varphi_h \dots$$
$$= i \cdot Z_L$$

-> mode converter

fundamental parameters

radiation characteristic



$$\lim_{r \to \infty} \frac{E}{H} = Z_{F0} = 120\dots$$

$$\underline{C}(\vartheta, \varphi) = \frac{E(\vartheta, \varphi \dots}{E_{max}}$$

$$C(\vartheta, \varphi) = \left| \frac{E(\vartheta,\varphi)}{E_{max}} \right|$$

$$= \sqrt{\frac{S(\vartheta, \varphi)}{S_{max}}}$$

gain & directivity

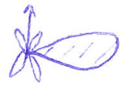
*) gain $G = \frac{S_{max}}{S_I} \Big|_{P_{t_0} = const}$ $\qquad S_I = \frac{P_{t_0}}{4\pi r^2}$

$$g = 10 \lg G$$

*) directivity $D = \frac{S_{max}}{S_I} \Big|_{P_t = const.}$

$$G = \eta \cdot D \qquad \eta = \frac{P_t}{P_{t_0}}$$

D from radiation characteristics:



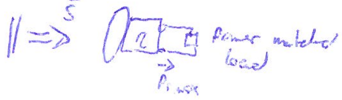
$$dA = r^2 d\vartheta\, d\varphi \sin\vartheta$$

$$P_t = \int S\, dA = \int_{\vartheta=0}^{\pi} \int_{\varphi=0}^{2\pi} S_{max}\, C^2(\vartheta, \varphi)\, r^2 d\vartheta d\varphi \sin\vartheta$$

$$\rightarrow D = \frac{S_{max}}{S_I} = \frac{4\pi}{\Omega_A} \qquad \Omega_A = \iint C^2 \sin\vartheta\, d\vartheta\, d\varphi$$

for highly directive antennas: $\Omega_A \approx \varphi_{3dB} \cdot \vartheta_{3dB}$

receiving antennas

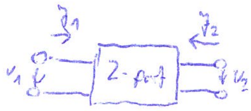


$$P_{r\,max} = S \cdot A_e \rightarrow A_e = \frac{P_{r\,max}}{S} \quad \text{... effective area}$$

$$A_0 = \frac{P_{r0\,max}}{S} \geq A_e \quad \text{... max. effective area}$$

$$\boxed{A_e = \frac{\lambda_0^2}{4\pi} \cdot G} \qquad \boxed{A_0 = \frac{\lambda_0^2}{4\pi} \cdot D}$$

reciprocity



$$\begin{pmatrix} U_1 \\ U_2 \end{pmatrix} = \|Z\| \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

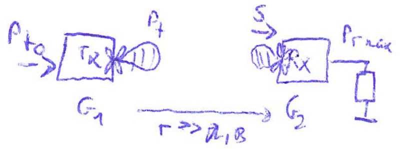
$$Z_{12} = Z_{21}$$

$\hookrightarrow C_{Rx} = C_{Tx} = C(\varphi, \vartheta)$

$\hookrightarrow G_{Rx} = G_{Tx} \quad (\rightarrow D_{Rx} = D_{Tx})$

wireless transmission

free space



$$S = \frac{P_{to}}{4\pi r^2} \cdot G_1$$

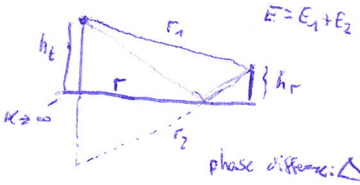
$$P_{r\,max} = S \cdot A_e = \frac{P_{to}}{4\pi r^2} \cdot \frac{\lambda_0^2}{4\pi} \cdot G_1 \cdot G_2$$

$$\rightarrow \boxed{\frac{P_{r\,max}}{P_{to}} = G_1 G_2 \left(\frac{\lambda_0}{4\pi r}\right)^2}$$

$$a = 10 \lg \left(\frac{4\pi r}{\lambda_0}\right)^2 - g_1 - g_2$$ ... isotropic radio link attenuation

over perfectly conducting ground

$E = E_1 + E_2$



$$r_1 = \sqrt{r^2 + (h_t - h_r)^2}$$

$$r_2 = \sqrt{r^2 + (h_t + h_r)^2}$$

phase difference: $\Delta r \approx \frac{2 h_t h_r}{r}$ $(h_t + h_r \ll r)$

$$\boxed{S_{max} = \frac{1}{2} \frac{|E_{max}|^2}{Z_{F_0}} = \frac{P_{to}}{4\pi r^2} \cdot G_T}$$ [far field]

$$\rightarrow |E_{max}| = \frac{1}{r} \sqrt{\frac{P_{to} Z_{F_0} G_T}{2\pi}}$$

$$\left[E \sim e^{-i\beta_0 r} = e^{-i \frac{2\pi}{\lambda_0} r}\right]$$

$$E = \underbrace{e^{i\chi} \sqrt{}}_{F} \cdot \frac{e^{-i\beta_0 r}}{r}$$

$e^{i\chi}$ ... phase of generator

$$E_\Sigma = E_{1max} + E_{2max} = F\left[\frac{e^{-i\beta_0 r_1}}{r_1} + \frac{e^{-i\beta_0 r_2}}{r_2}\right]$$

→ reflection factor $= -1$

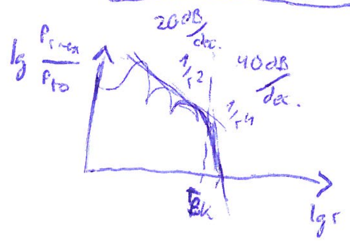
$$= F \frac{e^{-i\beta_0 r_1}}{r_1}\left[1 - \frac{r_1}{r_2} e^{-i\beta_0 (r_2 - r_1)}\right]$$

$r_1 \approx r_2 \approx r$

$$\approx j F e^{-i\beta_0 r} \frac{4\pi h_t h_r}{\lambda_0 r^2}$$

$$\rightarrow |E_\Sigma| = |F| \cdot \frac{4\pi h_t h_r}{\lambda_0 r^2} \longrightarrow S_{max} = \frac{P_{to} G_t 4\pi h_t^2 h_r^2}{\lambda_0^2 r^4}$$

$$\rightarrow \boxed{P_{r\,max} = \frac{P_{to} G_t G_r h_t^2 h_r^2}{r^4}}$$



$$\boxed{r_{Bk} \approx 8{,}4 \cdot \frac{h_t \cdot h_r}{\lambda_0}}$$ ... breakpoint distance

*) farthest point of min. attenuation: ⟷

reflection coeff. for grazing incidence: $\rho = -1$

→ 180° phase shift

constructive interference: $\Delta r \doteq (2n-1)\frac{\lambda_0}{2} = \frac{2 h_t h_r}{r}$

farthest point: $\Delta r = \frac{\lambda_0}{2}$

*) farthest point of max attenuation ($\Delta r = \lambda_0$)

$$|E_S| = |F|\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = |F|\left(\frac{1}{r_1} - \frac{1}{r_1 + \lambda_0}\right) = \sqrt{\frac{P_{to} Z_{F_0}}{2\pi}} \frac{\lambda_0}{r_1 (r_1 + \lambda_0)}$$

$$\approx \% \frac{\lambda_0}{r^2}$$

$$P_{r\,max} = S \cdot A_e = \frac{1}{2} \frac{|E_S|^2}{Z_{F_0}} \cdot \frac{\lambda_0^2}{4\pi} = \frac{P_{to}}{16\pi^2}\left(\frac{\lambda_0}{r}\right)^4$$

$$P_{r\,max\ free\ space} = P_{to}\left(\frac{\lambda_0}{4\pi r}\right)^2$$

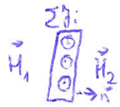
$$a = \left(\frac{r}{\lambda_0}\right)^2$$

In general: $\frac{P_{r\,max}}{P_{to}} \sim \frac{1}{r^n}$, $n = \begin{cases} 2 & \text{free space} \\ 2{,}7\text{–}3{,}5 & \text{urban area} \\ 3\text{–}5 & \text{shadowed ...} \end{cases}$

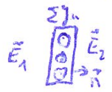
Summary RFT III

Antennas

Equivalent field sources



$$\oint \vec{H} d\vec{s} = \Sigma J_i \rightarrow \vec{J}_* = \vec{n} \times (\vec{H}_2 - \vec{H}_1)$$



$$\oint \vec{E} d\vec{s} \approx s(\vec{E}_2 - \vec{E}_1) = \underbrace{-j\omega\phi}_{J_m} \rightarrow E_2 - E_1 = -\frac{J_m}{s} = -\vec{M}_*$$

Uniqueness theorem & Huygens principle

Uniqueness theorem:

the electromagnetic field outside volume V in source-free space (with inner losses) is uniquely determined by E_t / H_t on the closed boundary surface O

Huygen's principle: each point on a primary wave front can be considered to be a new source of a secondary spherical wave

$\rightarrow$ electric & magnetic surface currents can be assumed to act as sources of secondary waves

Rad

Radiation from electromagnetic Sources, Hertzian vector

$$\text{rot}\, \vec{E}_2 = j\omega\mu_0 \vec{H}_2 + \vec{M}$$
$$\text{rot}\, \vec{H}_2 = j\omega\varepsilon_0 \vec{E}_2$$
$$\vec{E}_2 = -j\omega\mu_0 \,\text{rot}\, \vec{\Pi}_M \leftarrow \text{div}(\varepsilon_0 \vec{E}_2) = 0$$
$$\downarrow$$
$$\text{rot}\, \vec{H}_2 = \underbrace{\omega^2 \mu_0 \varepsilon_0}_{\beta_0^2} \,\text{rot}\, \vec{\Pi}_M$$
$$= \text{rot}(\beta_0^2 \vec{\Pi}_M + \text{grad div}\, \vec{\Pi}_M)$$
$$\rightarrow \boxed{H_2 = \beta_0^2 \vec{\Pi}_M + \text{grad div}\, \vec{\Pi}_M}$$
$$\Rightarrow \Delta \vec{\Pi}_M + \beta_0^2 \vec{\Pi}_M = \frac{-\vec{M}}{j\omega\mu_0}$$

solution

$$\boxed{\vec{\Pi}_M = \iiint_V M(\vec{r}\,') \frac{e^{-j\beta_0 |\vec{r} - \vec{r}\,'|}}{j\omega\mu_0 4\pi |\vec{r} - \vec{r}\,'|} dV}$$

Maxwell

$$\text{rot}\, \vec{H}_1 = j\omega\varepsilon_0 \vec{E}_1 + \vec{J}$$
$$\text{rot}\, \vec{E}_1 = -j\omega\mu_0 \vec{H}_1 + \vec{M}$$
$$\text{div}(\mu_0 \vec{H}) = 0 \qquad \xrightarrow{\text{div rot}(x) \equiv 0} H_1 = j\omega\varepsilon_0 \,\text{rot}\, \vec{\Pi}$$
$$\text{div}(\varepsilon_0 \vec{E}_1) = 0$$
$$\downarrow$$
$$\text{rot}\, \vec{E}_1 = \underbrace{\omega^2 \mu_0 \varepsilon_0}_{\beta_0^2} \,\text{rot}\, \vec{\Pi}$$

Laplace calibration

$$= \text{rot}(\beta_0^2 \vec{\Pi} + \underbrace{\text{grad div}\, \vec{\Pi}}_{=0})$$
$$\rightarrow \boxed{\vec{E}_1 = \beta_0^2 \vec{\Pi} + \text{grad div}\, \vec{\Pi}}$$
$$\Rightarrow \Delta \vec{\Pi} + \beta_0^2 \vec{\Pi} = \frac{-J}{j\omega\varepsilon_0} \quad \text{(cartesian)}$$
$$\Diamond \ldots \quad \text{(general)}$$

$\boxed{\Diamond A = \text{grad div}\, \vec{A} - \text{rot rot}}$

solution

$$\boxed{\vec{\Pi} = \iiint_V J(\vec{r}\,') \frac{e^{-j\beta_0 |\vec{r} - \vec{r}\,'|}}{j\omega\varepsilon_0 4\pi |\vec{r} - \vec{r}\,'|} dV'}$$

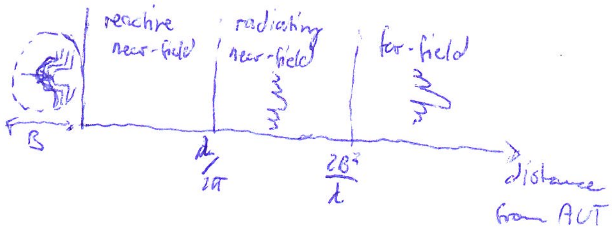
Farfield Approximation

$$|\vec{r}| \gg |\vec{r}\,'|, \lambda_0 \qquad \Delta r = \frac{\vec{r} \cdot \vec{r}\,'}{|\vec{r}|} = |\vec{r}\,'| \cos\alpha$$

$$|\vec{r} - \vec{r}\,'| \approx r$$
$$\rightarrow \vec{r} - \vec{r}\,' \approx r - \Delta r = r - r' \cdot r$$

$$\rightarrow \boxed{\vec{\Pi} = \frac{e^{-j\beta_0 r}}{j\omega\varepsilon_0 4\pi r} \iiint_V J(\vec{r}\,') e^{j\beta_0 \frac{\vec{r}\,' \cdot \vec{r}}{|\vec{r}|}} dV} \qquad \boxed{\vec{\Pi}_M = \frac{e^{-j\beta_0 r}}{j\omega\mu_0 4\pi r} \iiint_V M(\vec{r}\,') e^{j\beta_0 \frac{\vec{r}\,' \cdot \vec{r}}{r}}}$$

Antenna field zones:



Calculation of field strengths in the far field using spherical coordinates:

$\vec{\Pi} = \Pi_z \vec{e}_z$

$\downarrow$

$E_z = -E_{\vartheta} = \omega \varepsilon \beta_0 Z_{F_0} \Pi_z(x)$

*) Originating from electric sources: $\vec{H}_1 = j\omega\varepsilon_0 \,\mathrm{rot}\, \vec{\Pi}$

$\downarrow r \gg \lambda_0$

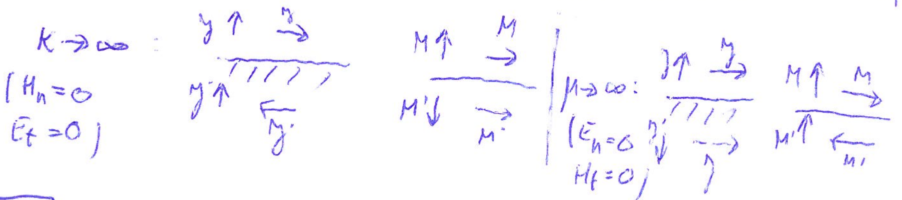
$$\boxed{\begin{aligned} &\underline{H}_{r_1} = 0 \\ &\underline{H}_{\vartheta_1} = -\omega\varepsilon_0\beta_0 \underline{\Pi}_{\varphi} \qquad \underline{E}_{\varphi_1} = -Z_{F_0}\underline{H}_{\vartheta_1} = \beta_0^2 \underline{\Pi}_{\varphi} \\ &\underline{H}_{\varphi_1} = \omega\varepsilon_0\beta_0 \underline{\Pi}_{\vartheta} \qquad \underline{E}_{\vartheta_1} = Z_{F_0}\underline{H}_{\varphi_1} = \beta_0^2 \underline{\Pi}_{\vartheta} \end{aligned}}$$

*) Originating from magnetic ": $\vec{\underline{E}}_2 = -j\omega\mu_0 \,\mathrm{rot}\, \vec{\underline{\Pi}}_m$

$\downarrow r \gg \lambda_0$

$$\boxed{\begin{aligned} &\underline{E}_{r_2} = 0 \\ &\underline{E}_{\vartheta_2} = \omega\mu_0\beta_0 \underline{\Pi}_{m\varphi} \qquad \underline{H}_{\varphi_2} = \frac{\underline{E}_{\vartheta_2}}{Z_{F_0}} = \beta_0 \Pi_{m\varphi} \\ &\underline{E}_{\varphi_2} = -\omega\mu_0\beta_0 \underline{\Pi}_{m\vartheta} \qquad \underline{H}_{\vartheta_2} = \frac{-\underline{E}_{\varphi_2}}{Z_{F_0}} = \beta_0 \Pi_{m\vartheta} \end{aligned}}$$

$\boxed{\text{Image theory}}$



$\boxed{\text{Application of Huygen's principle}}$

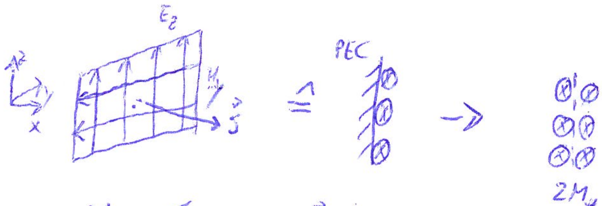
$\vec{J}_* = \vec{n} \times \vec{H}$

$\vec{M}_* = -n \times \vec{E}$

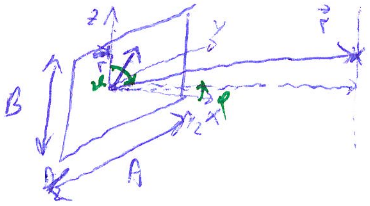
*) equivalent {electric / magnetic} surface current on {PEC / PMC} do not radia[te]

~~Appert~~

$\boxed{\text{Apertrue antennas}}$



$M_* = E$



$\vec{r}' = (0, y, z)$

$\vec{r} = (r\sin\vartheta\cos\varphi,\ r\sin\vartheta\sin\varphi,\ r\cos\vartheta)$

$\Delta r = |r'| \cdot |r| \cdot \cos\alpha$

$$\rightarrow \Pi_{my} = \frac{e^{-j\beta_0 r}}{j\omega\mu_0 4\pi r} \int_{-B/2}^{B/2}\int_{-A/2}^{A/2} 2 \cdot E(y,z)\, e^{-j\beta_0 (y r\sin\vartheta\sin\varphi + z\cos\vartheta)}\, dy$$

~~$E \neq E_0 \cdot G_y(\frac{y}{\lambda_0}) G_z(\frac{z}{\lambda})$~~ $\rightarrow$ ~~$= \frac{\lambda_0^2}{j\omega\mu_0 2\pi} \frac{e^{-j\beta r}}{r} E_0 \int G_y(\frac{y}{\lambda_0}) e^{j2\pi\sin}$~~

Antennas

Aperture antennas

$$\Pi_{my} = \frac{e^{-j\beta_0 r}}{j\omega\mu_0 4\pi r} \int_{-B/2}^{B/2}\int_{-A/2}^{A/2} \alpha E(y,z)\, e^{-j\beta_0 (y \sin\vartheta \sin\varphi + z\cos\vartheta)}\, dy\, dz$$

$E(y,z) = E_0\, G_y\left(\frac{y}{\lambda_0}\right) G_z\left(\frac{z}{\lambda_0}\right) \rightarrow$

$\beta_0 = \frac{2\pi}{\lambda_0}$

$$\boxed{\Pi_{my} = \frac{\lambda_0^2}{j\omega\mu_0 2\pi} \frac{e^{-j\beta_0 r}}{r} E_0 \int_{-A/2\lambda_0}^{A/2\lambda_0} G_y\left(\frac{y}{\lambda_0}\right) e^{j2\pi \sin\vartheta \sin\varphi \frac{y}{\lambda_0}}\, d\left(\frac{y}{\lambda_0}\right) \cdot \int_{-B/2\lambda_0}^{B/2\lambda_0} G_z\left(\frac{z}{\lambda_0}\right) e^{j2\pi\cos\vartheta \frac{z}{\lambda_0}} d\left(\frac{z}{\lambda_0}\right)}$$

aperture with uniform distribution: $G_y = G_z = 1$

"Fourier-Transform": $f' = \frac{y}{\lambda_0}, \frac{z}{\lambda_0}$ period $p \hat{=} \frac{A}{\lambda_0}, \frac{B}{\lambda_0}$

$t' = \sin\varphi \sin\vartheta, \cos\vartheta$

$$\rightarrow \Pi_{my} = \frac{A}{\lambda_0}\frac{B}{\lambda_0}\frac{\lambda_0^2}{j\omega\mu_0 2\pi} E_0 \frac{e^{-j\beta_0 r}}{r} \underbrace{\overbrace{si\left(\pi \frac{A}{\lambda_0} \sin\varphi \sin\vartheta\right)}^{si(1)} \overbrace{si\left(\pi \frac{B}{\lambda_0}\cos\vartheta\right)}^{si(2)}}_{C(\vartheta,\varphi)}$$

$$\rightarrow E_\vartheta = \omega\mu_0\beta_0 \Pi_{m\varphi}$$

$$= \omega\mu_0\beta_0 \Pi_{my} \cos\varphi$$

$$= \beta_0 \cdot \frac{AB}{2\pi j} E_0 \frac{e^{-j\beta_0 r}}{r}\left[\cos\varphi\, si(1)\, si(2)\right]$$

H-Plane: $\vartheta = 90°$

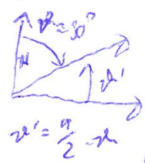
$$C_H(\varphi) = \underbrace{\cos\varphi}_{\approx 1}\, si\left(\pi\frac{A}{\lambda_0}\sin\varphi\right) \underbrace{si(0)}_{=1}$$

$$= si\left(\pi \frac{A}{\lambda_0}\sin\varphi\right)$$

$$\boxed{\begin{aligned}\varphi_{3dB} &= 2\arcsin\left(0{,}44\frac{\lambda_0}{A}\right)\\ &\approx 0{,}88\frac{\lambda_0}{A}\,[rad]\\ &= 50{,}4°\frac{\lambda_0}{A}\,[deg]\end{aligned}}$$

E-Plane: $C_E(\vartheta) = si\left(\pi\frac{B}{\lambda_0}\cos\vartheta\right)$

$$= si\left(\pi\frac{B}{\lambda_0}\sin\vartheta'\right)$$



$$\boxed{\begin{aligned}\vartheta'_{3dB} &= 2\arcsin\left(0{,}44\frac{\lambda_0}{B}\right)\\ &\approx 50{,}4°\frac{\lambda_0}{B}\end{aligned}}$$

Directivity & gain

$$P_t = \frac{1}{2}\frac{|E_0|^2}{Z_{F0}} AB$$

$$S_i = \frac{P_t}{4\pi r^2} = \frac{1}{2}\frac{|E_0|^2 AB}{4\pi r^2 Z_{F0}}$$

$$D = \frac{S_{max}}{S_i} \qquad \boxed{S_{max} = \frac{1}{2}\frac{|E_{\vartheta,max}|^2}{Z_{F0}} = \frac{1}{2}\frac{|E_0|^2 (AB)^2}{\lambda_0^2 r^2 Z_{F0}}}$$

$$= \frac{4\pi}{\lambda_0^2} AB$$

Receiving case: max. aperture: $\boxed{A_0} = q \cdot A_{geo} \overset{\text{uniform distr.}}{=} A \cdot B$ ($q = 100\%$) aperture efficiency

$$= \boxed{D \cdot \frac{\lambda_0^2}{4\pi}}$$

gain: $G = \eta \cdot D \Rightarrow A_e = G \cdot \frac{\lambda_0^2}{4\pi}$ effective area

Horn Antenna: infinite length:

$$E_{\vartheta} = \frac{2}{\pi}\frac{ABE_0\, e^{-j\beta_0 r}}{j\lambda_0 r}\, C(\vartheta,\varphi)$$

$$C(\vartheta,\varphi) = \cos\varphi \frac{\cos\left(\pi\frac{A}{\lambda_0}\sin\vartheta\sin\varphi\right)}{1-\left(2\frac{A}{\lambda_0}\sin\vartheta\sin\varphi\right)^2}\underbrace{\mathrm{si}\left(\frac{B}{\lambda_0}\cos\vartheta\right)}_{\text{unchanged}}$$

H-Plane: $\varphi_{3dB} \approx \frac{68{,}9\,\lambda_0}{A}$

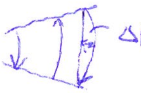
$\vartheta_{3dB} \approx \frac{50{,}4\,\lambda_0}{B}$

$$D = \frac{S_{max}}{S_i} = \frac{\left(\frac{2}{\pi}\right)^2}{1/2} D_{Uni} = \frac{8}{\pi^2} D_{Uni} \approx 0{,}81 D_{Uni}$$

$q = 81\%$

$q_{SL} = 20\log\frac{1}{0{,}07} \approx 23\,dB$

Pyramidical horn:



$\Delta\beta$ phase diff.

→ not a plane wave

~~Rudges~~ horn → $\Delta\beta$ ↑

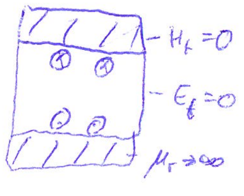
→ compromiss: optimum between length of Horn and phase diff.

→ Maximum gain horn

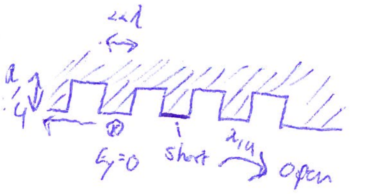
$$L_H^2 = \left(\frac{A}{2}\right)^2 + L^2\left(\frac{1}{1-\frac{a}{A}}\right)^2$$

$$L_E^2 = \left(\frac{B}{2}\right)^2 + L^2\left(\frac{1}{1-\frac{b}{B}}\right)^2$$

Corrugated Horn



Idea: introducing grooves:

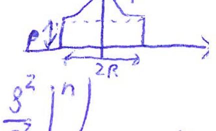


⊕ very low sidelobes $\qquad E_x: Z_\square = \infty$

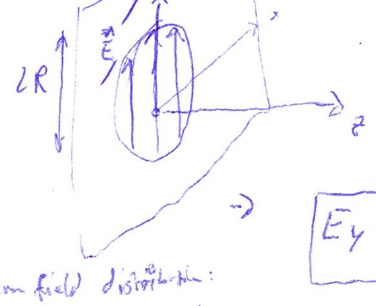
⊕ extreme good performance $\qquad E_y: Z_\square = 0$

⊖ not practical

Circular Apertures



$$E_y(\varrho) = E_0\left(p + \left(1-\frac{\varrho^2}{R^2}\right)^n\right)$$



$\cos(\varphi-\psi)$

$$\Pi_{mx} = \frac{2E_0\, e^{-j\beta_0 r}}{j\omega\mu_0 4\pi r}\int_{\varphi=0}^{2\pi}\int_{\rho=0}^{R}\left(p+\left(1-\left(\frac{\varrho}{R}\right)^2\right)^n\right) e^{j\beta_0\varrho\sin\vartheta\cos(\varphi-\psi)}\,\varrho\, d\varrho\, d\varphi$$

→

$$\boxed{E_y = \left(p + \frac{1}{n+1}\right)\cdot\frac{jE_0\beta_0 R^2}{2}\frac{e^{-j\beta_0 r}}{r}\, C(\vartheta)}$$

uniform field distribution:
$p = 0,\ n = 0$

Antennas

Aperture antennas

Circular apertures

$$\boxed{C(\vartheta) = \frac{1}{p + \frac{1}{n+1}} \left[p \cdot \frac{J_1(\beta_0 R \sin\vartheta)}{\frac{\beta_0 R \sin\vartheta}{2}} + \frac{n!\, J_{n+1}(\beta_0 R \sin\vartheta)}{\left(\frac{\beta_0 R \sin\vartheta}{2}\right)^{n+1}} \right]}$$

aperture distribution: $\frac{E(\rho)}{E_0} = p + \left(1 - \frac{\rho^2}{R^2}\right)^n$

aperture efficiency: $q = \frac{\left(p + \frac{1}{n+1}\right)^2}{p^2 + \frac{2p}{n+1} + \frac{1}{2n+1}}$

uniform field distribution: $(p=0, n=0)$

$$C(\vartheta) = 2 \cdot \frac{J_1(\pi \cdot U)}{\pi \cdot U}$$

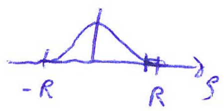
with $U = \frac{\beta_0}{\pi} R\sin\vartheta = \frac{2R}{\lambda_0}\sin\vartheta$

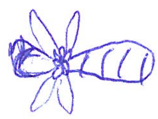
$$\left[J_n(x) \overset{x \ll 1}{\approx} \frac{(x/2)^n}{n!} \right]$$

$$a_{SL} = 17{,}7\,dB$$

$$\vartheta_{3dB} = 57^\circ \cdot \frac{\lambda_0}{2R}$$

Heavily tapered $(n \hat{=} 2)$





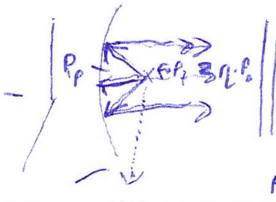
⊕ reduced sidelobe level

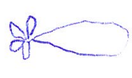
⊖ ~~for~~ lightly directivity → big horn antennas (all) → not practical

$\vartheta_{3dB} = 84^\circ \frac{\lambda_0}{2R}$

$a_{SL} \approx 30{,}6\,dB$ → sidelob att. is more important than small bea[m]

Reflector Antennas ~~motivation~~: in-phase illumination ~~wave~~ with of large aperture

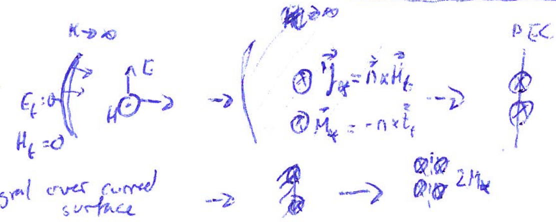




also radiation in backside direction

spill over efficiency: $q_S = \frac{P_P}{P_t} \approx 0{,}98$ → Method of Moments (MoM) is better to describe (→ compute

Virtual Aperture Method:



⊖: integral over curved surface

Wire antennas

(aerials)

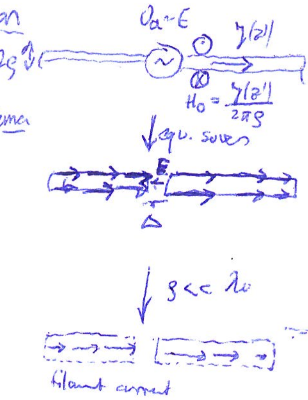
2ρ $L \gg \rho$

$\rho \gg \lambda_0$

$\int \vec{J}_A(z') e^{j\beta_0 \frac{\vec{r}'\cdot\vec{r}}{r}} dV'$

→ MoM (computer)

Radiation model of a wire antenna



$J(z = \frac{L}{2}) = 0$

$\vec{J}_F = -\vec{n} \times \vec{H}_0$

$\vec{M}_F = \vec{M}_M = \vec{n} \times \vec{E}$ | neglected (Δ very small)

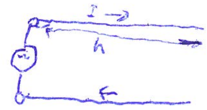
$J_M = \int M_F \, dz'$

$= \int E \, dz' = U_a$

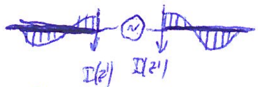
$J(z') = 2\pi\rho \, J_F'$

Current distribution on symmetrical antennas using TLT (transmission line theory)

transmission line:



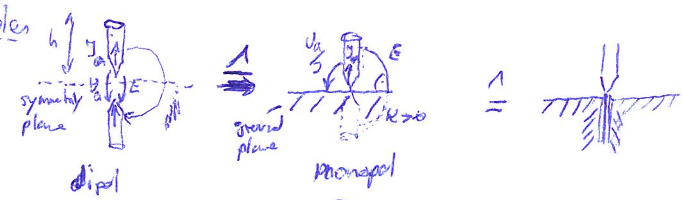
↓



≙ straight dipole

$I(z') = I_{max} \sin[\beta_0 (h - z')]$

straight (linear) dipoles and monopoles



$Z_d = \frac{U_a}{I_a}$ $\qquad Z_m = \frac{Z_d}{2}$

$h = \frac{\lambda_0}{4}$ $\qquad \frac{\lambda_0}{2\rho}$ ... Schlankheitsgrad

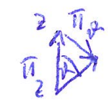
$\rightarrow h = \frac{\lambda_0}{4}(1-k) \quad , \quad k = k\left(\frac{\lambda_0}{2\rho}\right)$

Farfield Integration over straight dipole: (Radiation)

$\vec{\Pi}_z = \frac{e^{-j\beta_0 r}}{j\omega\varepsilon_0 4\pi r} \int \vec{J} e^{j\beta_0 \frac{\vec{r}'\cdot\vec{r}}{r}} dV'$

simplifications:

$\forall \; dz'$ — dA $\quad dV' = dA \cdot dz'$



$\Pi_\vartheta = -\Pi_z \sin\vartheta$

$\forall$ loc. vector: $\vec{r}' = (0, 0, z')$

$\forall$ obs. $\nearrow$: $\vec{r} = \begin{pmatrix} r\sin\vartheta\cos\varphi \\ r\sin\vartheta\sin\varphi \\ r\cos\vartheta \end{pmatrix}$

$\forall \; \rightarrow \frac{\vec{r}'\cdot\vec{r}}{r} = z\cos\vartheta$

$\Rightarrow \Pi_z = \frac{e^{-j\beta_0 r}}{j\omega\varepsilon_0 4\pi r} J_A \underbrace{\int_{-\frac{\lambda_0}{4}}^{\frac{\lambda_0}{4}} \cos(\beta_0 z') e^{j\beta_0 z' \cos\vartheta}}_{\frac{2\cos(\frac{\pi}{2}\cos\vartheta)}{\beta_0 \sin^2\vartheta}}$

$H_{\varphi_1} = \omega\varepsilon_0\beta_0 \Pi_\vartheta \; ; \; E_{\vartheta_1} = Z_{F0} \cdot H_{\varphi_1}$

$\boxed{E_\vartheta = j J_A \cdot Z_{F0} \frac{e^{-j\beta_0 r}}{2\pi r} \cdot C(\vartheta)} \quad C(\vartheta) = \left|\frac{\cos(\frac{\pi}{2}\cos\vartheta)}{\sin\vartheta}\right|$

Summary RFT III

Antennas — Wire Antennas — Straight wire dipoles and monopoles

Radiation

$$S = \frac{1}{2}\frac{|E|^2}{Z_{F0}}$$

$$P_t = \int \frac{1}{2}\frac{|E_\vartheta|^2}{Z_{F0}}$$

$$\approx \frac{1}{2}|J_A|^2 \cdot R_r$$

$$\boxed{R_{r0} = \frac{Z_{F0}}{\pi} \cdot 0{,}61 \approx 73\,\Omega} \rightarrow \boxed{R_{rm} = 36{,}5\,\Omega}$$

$\eta \approx 100\,\%$

$$\boxed{D} = \frac{S_{max}}{S_i} = \frac{S_{max}}{\frac{P_{r0}}{4\pi r^2}} \approx \boxed{1{,}64 \hat{=} 2{,}15\,dBi}$$

$D = G$, $1{,}64 = g$

Fullwave dipole

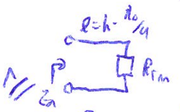
far field

$H = 0$ for $\vartheta = 90°$
$E = 0$ for $\vartheta = 90°$
(current not in phase → destruction)

$$I(z') = I_{max}\sin\left[\beta_0(h - |z'|)\right]$$

$$C(\vartheta) = \left|\frac{\cos(\beta_0 h \cos\vartheta) - \cos(\beta_0 h)}{\sin\vartheta\left[1 - \cos(\beta_0 h)\right]}\right|$$

General length dipol impedance: use TLT



1) $h > \frac{\lambda_0}{4}$:

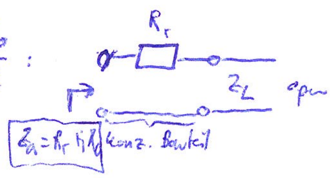
$$\boxed{Z_L \approx 120\,\Omega\left[\ln\left(\frac{h}{g}\right) - 0{,}8\right]}$$

*) R_{rm} aus Diagramm

*) S.D.: R_m einzeichnen → um $h - \frac{\lambda_0}{4}$ drehen

$$Z_a = \frac{1 + j\frac{Z_L}{R_{rm}}\tan\left[\beta_0\left(h - \frac{\lambda_0}{4}\right)\right]}{1 + j\frac{R_{rm}}{Z_L}\tan\left[\beta_0\left(h - \frac{\lambda_0}{4}\right)\right]}$$

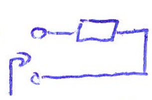
2) $h < \frac{\lambda_0}{4}$:



*) R_r aus Diagramm

*) $X_a = Z_L \frac{1}{\tan\beta h}$

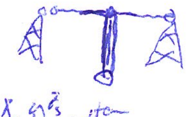
3) $h = \frac{\lambda_0}{4}$:

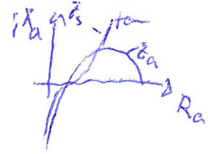


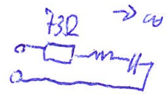
$Z_a = R_r = 73\,\Omega$

Design & Feeding of Dipol Antennas

smallband dipoles



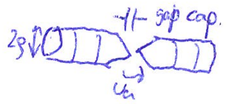




feeding orthogonal to axes of dipol
→ no disturbance

broadband dipoles

$\rho\uparrow$ (reduction of Z_L)
= thick dipole



conical design to avoid shunt capaci[…]

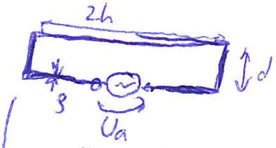
very high capacitance

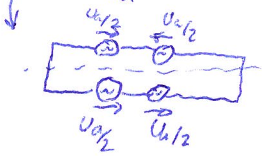
folded dipole

for low frequ.: just a loop
for high frequ.: very effici[…] antenna

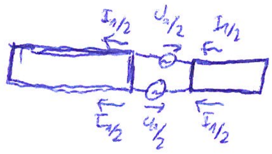
for lightening protection (low frequ.)

even/odd mode analysis:





even mode:

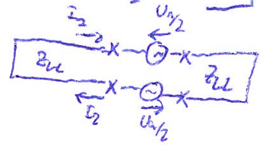


~~equ.~~ diamete[…] of equ. dipol

$r_e = \sqrt{d\rho}$

$$\boxed{Z_d = \frac{U_a/2}{2\cdot\frac{I_1}{2}} = \frac{U_a}{2I_1}}$$

odd mode:



$\hat{=}$

$$Z_{TL} = jZ_L \tan(\beta_0 h)$$

$$I_a = \frac{I_1}{2} + I_2$$

$$Z_A = \frac{U_a}{I_a} = \frac{U_a}{\frac{U_a}{2\cdot 2Z_d} + \underbrace{\frac{U_a}{2Z_{TL}}}_{=0 \text{ for } h=\lambda/4}} = 4Z_d \approx 29[…]$$

Antennas

Wire antennas

Balun

(balc

a) 

Small displacement currents
→ very small influence on radiation

b) 

sheath currents → cable radiates
(length of cable has influence

→ different radiation characteristics

↓

balun to compensate

Electrical small antennas (elementary dipoles)

$h \ll \lambda_0$ →  current distribution ~ triangular ($x \approx \sin x$ for $x \ll 1$)

↓ assume uniform current distribution of equivalent length l_e

Hertzian: $\vec{\Pi}_m$

electric

Hertzian Dipol: a) $\vec{\Pi} = \frac{I_a \cdot l_e}{j\omega\varepsilon_0 4\pi} \frac{e^{-j\beta_0 r}}{r}$ $l_e = h$

↓

$$E_r = Z_{F0} \frac{I_a l_e}{2\pi r^2} \cos\vartheta \left(1 + \frac{1}{j\beta_0 r}\right) e^{-j\beta_0 r}$$

$$E_\vartheta = j Z_{F0} \frac{\beta_0 I_a l_e}{4\pi r} \sin\vartheta \left(1 + \frac{1}{j\beta_0 r} - \frac{1}{(\beta_0 r)^2}\right) e^{-j\beta_0 r}$$

$$H_\varphi = j \frac{\beta_0 I_a l_e}{4\pi r} \sin\vartheta \left(1 + \frac{1}{j\beta_0 r}\right) e^{-j\beta_0 r}$$

$D_H = 1{,}5 \hat{=} d = 1{,}8\,dB$
Hertzscher Dipol

$(D_{\lambda/4} = 1{,}64 = 2{,}15\,dB)$

$$\boxed{R_r = 80\pi^2 \frac{l_e^2}{\lambda_0^2}}$$

far field: $\boxed{E_\vartheta = H_\varphi Z_{F0} = j Z_{F0} \frac{\beta_0 I_a l_e}{4\pi r} \sin\vartheta\, e^{-j\beta_0 r}}$

equivalence: $I_a \cdot l_e = j\omega\varepsilon_0 I_m \cdot A$

(magnetic)

b) far field: $\boxed{H_\vartheta = -\frac{E_\varphi}{Z_{F0}} = \frac{j\beta_0 I_m \Delta}{4\pi r} \sin\vartheta\, e^{-j\beta_0 r}}$

equivalence: $I_m \Delta = j\omega\mu_0 I \cdot A$

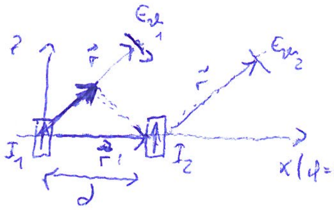
(i.e. loop antennas:

Receiving antennas
(considering dipoles, monopoles)

reciprocity: A_e, l_e, Z_a, D, G the same

$\boxed{\text{Antenna arrays}}$

Radiation characteristic, array factor



$$I_1 = I_2 \cdot e^{j\psi} \qquad \vec{r} = (r\sin\vartheta\cos\varphi,\ r\sin\vartheta\sin\varphi,\ r\cos\vartheta)$$

$$|\Delta\vec{r}| = \frac{\vec{r}\cdot\vec{r}'}{r} = d\sin\vartheta\cos\varphi$$

$$\underline{E}_{\vartheta_1} = \underline{E}_{\vartheta_2}\cdot e^{j\delta} \qquad \boxed{\delta = \psi - \beta_0 d\sin\vartheta\cos\varphi}$$

$$|E_{\vartheta_a}| = \underbrace{2\,|E_{\vartheta_2}|}_{|E_{\vartheta_2}|_{max}\cdot C_s(\vartheta,\varphi)}\cdot\underbrace{\left|\cos\frac{\delta}{2}\right|}_{C_g(\vartheta,\varphi)}$$

$$\boxed{C_a(\vartheta,\varphi) = C_S(\vartheta,\varphi)\cdot C_g(\vartheta,\varphi)}$$

Phased Array

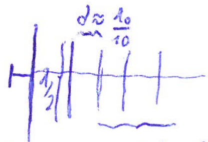
$$I_1 = I_2 e^{+j\psi} \qquad I_3 = I_2\cdot e^{j\psi} \qquad \delta = \psi - \beta_0 d\cos\varphi$$

$$E_{\vartheta\,array} = E_{\vartheta_1}(1 + e^{j\delta} + e^{j2\delta} + \ldots)$$
$$= E_{\vartheta_1}\sum_{i=0}^{n-1} e^{ji\delta}$$
$$= E_{\vartheta_1}\frac{e^{jn\delta}-1}{e^{j\delta}-1}$$

$$C_{g_r} = \frac{|E_{\vartheta_a}|}{n\cdot|E_{\vartheta_1}|} = \left|\frac{\sin\left(\frac{n\delta}{2}\right)}{n\cdot\sin\left(\frac{\delta}{2}\right)}\right| \qquad (\text{max. if } \delta\to 0)$$

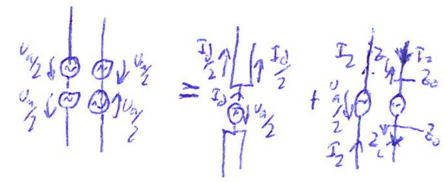
arrays with parasitic elements

Yagi-Uda-Antenna:



$>\lambda/2 \hat{=}$ reflector $\quad$ $\leq \lambda/2 \hat{=}$ director

even-/odd mode analysis:



"S.D."

$$I_d = \frac{U_a}{Z_d} \qquad I_2 = \frac{U_a}{2Z_{op}}$$

$$\frac{I_p}{I_a} = \frac{Z_{op}/2Z_d - 1}{Z_{op}/2Z_d + 1} \quad \begin{cases} I_a = \frac{I_d}{2} + I_2 = \frac{U_0}{2}\left(\frac{1}{Z_d} + \ldots\right) \\ I_p = \frac{I_d}{2} - I_2 = \frac{U_0}{2}\left(\frac{1}{Z_0} - \ldots\right) \end{cases}$$

(parasitic currents related to I_a)

max. effect of director: $I_p = I_a\cdot e^{-j\beta_0 d}$

reflector: $I_p = -I_a e^{-j\beta_0 d}$
$= I_a e^{j(\pi - \beta_0 d)}$

Summary RFT III

Electronic noise

Introduction:

$$\frac{S}{N} = \frac{P_S}{P_N} \quad \text{... signal to noise ratio}$$

$$N_L = 10 \lg \frac{P_S}{P_N} \quad \text{... noise level}$$

Properties of noise signals

mean value: $u^{\Delta t} = \frac{1}{\Delta t} \int_{t_0}^{t_0+\Delta t} u(t)\, dt \xrightarrow{\Delta t \to \infty} 0$

mean square: $(u^{\Delta t})^2 = \frac{1}{\Delta t} \int_{t_0}^{t_0+\Delta t} u^2(t)\, dt \xrightarrow{\Delta t \to \infty} > 0$

root mean square: $\tilde{u} = \sqrt{\overline{u^2}}$

Gaussian PDF $\quad p^*(u_0) = \frac{e^{-\frac{1}{2}\left(\frac{u_0}{\tilde{u}}\right)^2}}{\sqrt{2\pi\, \tilde{u}^2}}$

$$P^*(u_1) = \int_{u_0=0}^{u_1} p^*(u_0)\, du_0 = \Phi^*\left(\frac{u_1}{\tilde{u}}\right) \approx \frac{1}{2} - \frac{e^{-\frac{1}{2}\frac{u_1^2}{\tilde{u}^2}}}{\sqrt{2\pi}\, \frac{u_1}{\sqrt{\tilde{u}^2}}}$$

$|u_1| > 2\sqrt{\tilde{u}^2}$

$[\Phi^*(-x) = \frac{1}{2} - \Phi^*(x)]$

Dealing with noise signals

Correlation $\quad \rho(\tau) = \overline{u(t) \cdot u(t+\tau)} \quad$ ... autocorrelation function

$$W_U(f) = 2 \int_{-\infty}^{\infty} \rho(\tau) \cos(2\pi f \tau)\, d\tau \quad \text{... power spectral density function}$$

$$\rho(\tau) = \int_0^{\infty} W_U(f) \cos(2\pi f \tau)\, df$$

$$\underline{W}_{U_{12}}(f) = 2 \int_{\mathbb{R}} \rho_{12}(\tau)\, e^{-j2\pi f \tau}\, d\tau$$

$$\boxed{\underline{\gamma} = \frac{\underline{W}_{U_{12}}(f)}{\sqrt{W_{U_1}}\sqrt{W_{U_2}}}} \quad \text{... correlation coeff.}$$

$$\rho_{12}(\tau) = \int_0^{\infty} \underline{W}_{U_{12}}(f)\, e^{j2\pi f \tau}\, df \quad \text{... cross correlation function}$$

Noise uncorrelated: $\quad \gamma = \lim_{n \to \infty} \left\{ \frac{1}{n} \sum_i \gamma_i \right\} = 0$

$\tilde{U}_{N1}$, $\tilde{U}_{N2}$ $\rightarrow$ $\tilde{U}_{NS}$

$$\boxed{|\tilde{U}_{NS}|^2 = |\tilde{U}_{N1}|^2 + |\tilde{U}_{N2}|^2}$$

Fully correlated: $\quad \boxed{\tilde{U}_{NS} = \hat{U}_1 + \hat{U}_2} \quad |\gamma| = 1 \quad (= e^{-j(\varphi_1 - \varphi_2)})$

(fixed phase relationship)

Particular correlated:

$$|\tilde{U}_1| = \sqrt{|\tilde{U}_{1U}|^2 + |U_{1C}|^2}$$

$$|\tilde{U}_{NS}| = \sqrt{|\tilde{U}_{1C} + \tilde{U}_2|^2 + |\tilde{U}_{1U}|^2}$$

Thermal noise of resistors

$R \;\hat{=}\; \tilde{U}_N \;\hat{=}\; \tilde{I}_N \; G \quad |\tilde{U}_N|^2 = W_{tU} \cdot \Delta f$

(*) real: $W_{tU} = \frac{h \cdot f}{kT} \cdot \frac{4kTR}{e^{hf/kT} - 1}$

$f < 3\,\text{GHz}$: white noise

$W_{ti} = \frac{1}{R^2} \cdot W_{tU} \qquad W_{tU} = 4kTR \qquad k = 1{,}38 \cdot 10^{-23} \frac{\text{J}}{\text{K}}$

$\Rightarrow \boxed{|U_N| = \sqrt{4kTR\Delta f}}$ (*)

$$\boxed{\text{effective noise bandwidth: } \Delta f = \int_0^{\infty} \frac{W_{tU}}{W_{tU_{max}}}\, df}$$

$T_0 = 293\,\text{K}$: $\boxed{|\tilde{U}_N|[\mu V] = 4\sqrt{R[k\Omega]} \cdot \sqrt{\Delta f[\text{MHz}]}}$

Available noise power

$$P_{N_{max}} = \left(\frac{|\tilde{U}_N|}{2}\right)^2 \cdot \frac{1}{R} = kT\Delta f$$

$$P_{N_{max}}[W] = 4 \cdot 10^{-21}\, \Delta f[\text{Hz}] \cdot \frac{T}{T_0}$$

Noise of Twoports

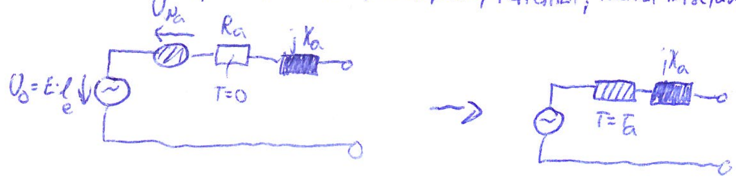
Zusatz Landstorfer:

Transmission over linear networks $\quad W_{U_2}(f) = |A(f)|^2 \cdot W_{U_1}(f)$

eff. noise bandwidth: $\Delta f = \int_0^{\infty} |A(f)|^2\, df$

Noise of receiving antennas

$\tilde{U}_{N_a}$ → atmospheric noise, ionospheric, terrestial, thermal noise (antenna losses), heat radiation of earth

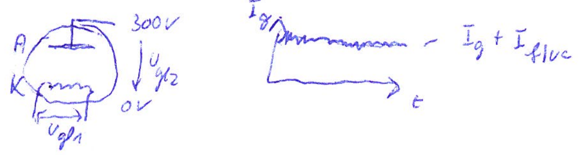


2^{nd} principle of thermodynamics: $T_a = T$ (room temperature)

Shot noise

$q_e = 1{,}6 \cdot 10^{-19}\,As$

Schottky equation:



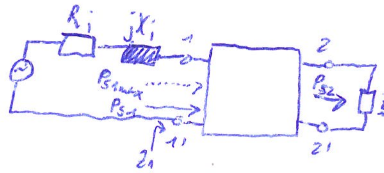
$$W_{is} = 2 \cdot q_e \cdot I_{g}$$

(main generator noise in electronic circuits)

$$|I_s|^2 = 2 q_e I_{g1} \cdot \Delta f$$

Noise of Twoports

Transduced power gain + extended noise figure



$\frac{S}{N}\Big|_{1-1'} > \frac{S}{N}\Big|_{2-2'}$

power match: $Z_1 = Z_i^* \rightarrow P_{S_1} = P_{S_1max}$ (else: $P_{S_1} < P_{S_1max}$)

transducer power gain: $g_T = \frac{P_{S_2}}{P_{S_1max}} = |A_B|^2$ → eff. transmission factor

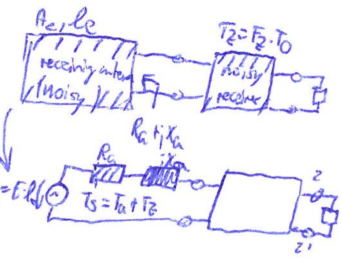
$P_{N_1max} = k T_0 \Delta f$ ... available noise power

$F = 1 + F_z = \frac{T_0 + T_z}{T_0}$ ← $P'_{N_1max} = P_{N_1max} + \Delta P_{N_1max} = k T_0 (1 + F_z) \Delta f$

$F_z = \frac{T_z}{T_0}$ .... zusätzliche Rauschzahl

$$\left.\begin{aligned} P_{N_2} &= g_T \cdot P'_{N_1max} \\ P_{S_2} &= g_T \cdot P_{S_1max} \end{aligned}\right\} \frac{P_{S_2}}{P_{N_2}} = \frac{P_{S_1max}}{k T_0 (1+F_z) \Delta f} \overset{!}{=} \max \;\downarrow\; \Delta f \downarrow$$

System noise temperature



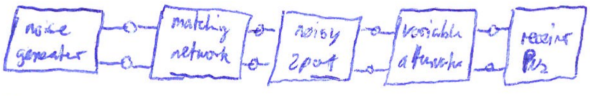
$$P_{S_1max} = S \cdot A_e = \frac{1}{2} \frac{|E|^2 l_e^2}{4 R_a}$$

$$P'_{N_1max} = k T_S \Delta f$$

advantage of 3dB-method:

no absolute scale of power meter needed

Measurement of noise figure of twoport



1.) noise gen. off, cold: $g_{v1} = 1 \triangleq 0\,dB$ ... Dämpfungsglied

2.) noise gen. on, hot: $g_{v2} = \frac{1}{2} \triangleq 3\,dB$

$P_{N_2} = (m k T_0 \Delta f + F_z k T_0 \Delta f) \frac{g_T}{2}$ ←

$P_{N_2} = (k T_0 \Delta f + F_z k T_0 \Delta f) g_T$

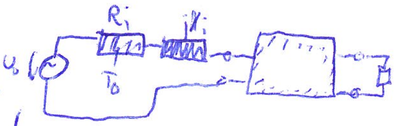
$P_{N_2,1} \overset{!}{=} P_{N_2,2} \rightarrow 1 + F_z = \frac{1}{2}(m + F_z)$

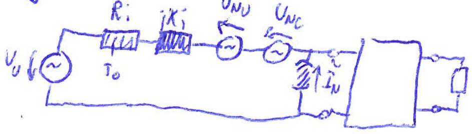
$\rightarrow F = F_z + 1 = m - 1$

Electronic noise

Noise of 2-ports

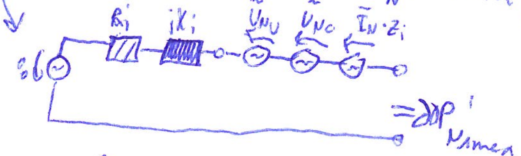
Noise modelling





$\tilde{U}_{NC} = \tilde{I}_N \cdot Z_C \qquad Z_C = R_C + jX_C$ ... correlation

$|U_{NU}|^2 = 4kT_0 R_n \Delta f \qquad |\tilde{I}_N|^2 = 4kT_0 G_n \Delta f$



$$|\tilde{U}_N|^2 = |\tilde{I}_N(Z_i + Z_C)|^2 + |\tilde{U}_{NU}|^2$$
$$= 4kT_0\Delta f\left[G_n \cdot |Z_i + Z_C|^2 + R_n\right]$$

$$\Delta P'_{N,1,max} = \frac{|\tilde{U}_N|^2}{4R_i} = F_Z kT\Delta f \qquad \text{with } F_Z = \frac{G_n}{R_i}\cdot|Z_i + Z_C|^2 + \frac{R_n}{R_i}$$

($\to$ minimum)

$\min\{F_Z\}$ for $Z_i = Z_{opt} = R_{opt} + jX_{opt}$

$$\to \boxed{R_{opt} = \sqrt{\frac{R_n}{G_n} + R_C^2}} \qquad \boxed{X_{opt} = -X_C}$$

$$\to F_{Z,min} = 2\left[G_n R_C + \sqrt{G_n R_n + (G_n R_C)^2}\right]$$

Circles of constant noise figure

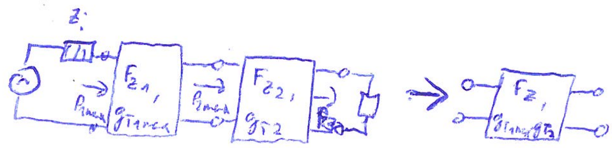
↓ recopy p. 7d

with $\varrho = \frac{Z_i - Z_{opt}}{Z_i + Z_{opt}}$, $\Delta F = G_n \cdot R_{opt}$

$$\hookrightarrow F_Z = F_{Z,min} + \Delta F \frac{4|\varrho|^2}{1-|\varrho|^2}$$

$$|\varrho| = \sqrt{\frac{(F_Z - F_{min})}{F_Z + 4\Delta F - F_{Z,min}}}$$

Cascaded Noisy 2-ports



$$P_{N3} = kT_0\Delta f\left[\{(1+F_{Z1})g_{T1max} + F_{Z2}\}g_{T2}\right]$$
$$= kT_0\Delta f\,(1+F_Z)\cdot g_{T1max}\cdot g_{T2}$$

a) F_{Z1} small

b) g_{T1max} large

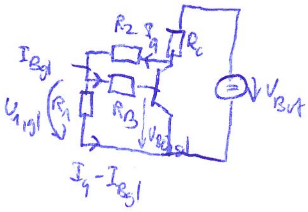
$\leftarrow \boxed{F_Z = F_{Z1} + \frac{F_{Z2}}{g_{T1max}}}$

noise measure: $M_i = \frac{F_{Zi}}{1 - 1/g_{Timax}}$

n-port: $\boxed{F_z = F_{z_1} + \frac{F_{z2}}{g_{T_1 max}} + \dots + \frac{F_{z_n}}{\prod_{i=1}^{n-1} g_{T_i max}}}$

RF 2-port amplifiers

Biasing



$U_{1gl} \approx 0{,}15\, U_{Bat}$ (at least $> 2 U_{BEgl}$)

$U_{BEgl} = 0{,}7V$

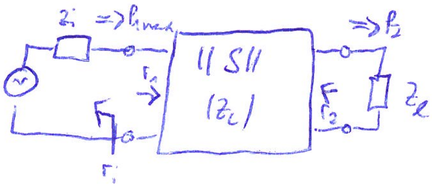
$I_q \geq 10\, I_{Bgl}$; $I_{Bgl} = \frac{I_{Cgl}}{B}$

Stability

$Re\{z_1\} \geq 0 \quad |r_1| \leq 1$

$Re\{z_2\} \geq 0 \quad |r_2| \leq 1$

Design



$r_i = \frac{z_i - z_L}{z_i + z_L} \qquad r_e = \frac{z_e - z_L}{z_e + z_L}$

$r_1 = S_{11} + \frac{S_{12} S_{21} r_e}{1 - S_{22} r_e} \qquad r_2 = S_{22} + \frac{S_{12} S_{21} r_i}{1 - S_{11} r_i}$

$g_T = \frac{P_2}{P_{1max}} = \frac{|S_{21}|^2 (1-|r_i|^2)(1-|r_e|^2)}{|(1-S_{11} r_i)(1-S_{22} r_e) - S_{12} S_{21} r_i r_e|^2}$

stability circles

*) load side: $|r_1| = 1$

$\rightsquigarrow \left| S_{11} + \frac{S_{12} S_{21} r_e}{1 - S_{22} r_e} \right| = 1$

if $|S_{11}| > 1$: inside circle stable

$|S_{11}| < 1$: outside circle stable

(reason: $r_e = 0 \Rightarrow r_1 = S_{11}$

$\Rightarrow \boxed{\sigma_e = \frac{(S_{22} - \det \underline{S} \cdot \underline{S}_{11}^*)^*}{|S_{22}|^2 - |\det \underline{S}|^2} \quad \varrho_e = \left| \frac{S_{12} S_{21}}{|S_{22}|^2 - |\det \underline{S}|^2} \right|}$

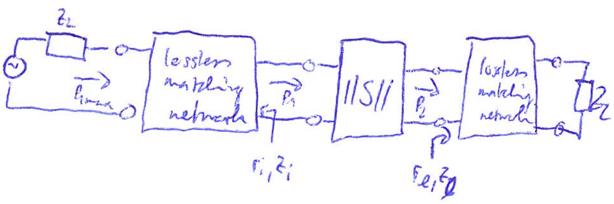
*) generator side: $|r_2| = 1$ $\quad \sigma_i = \frac{(S_{11} - \det S \cdot S_{22}^*)^*}{|S_{11}|^2 - |\det S|^2}$

$\varrho_i = \left| \frac{S_{12} S_{21}}{|S_{11}|^2 - |\det S|^2} \right|$

$\boxed{1 < K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\det S|^2}{2 |S_{12}| |S_{21}|} \quad \& \; |\det S| < 1}$ $\leftarrow$ Unconditional stability: *) $|\sigma_e| - \varrho_e > 1$, $|S_{11}| < 1$ & *) $|\sigma_i| - \varrho_i > 1$, $|S_{22}| < 1$

Unilateral design

$S_{12} \equiv 0$



$$g_{TU} = \frac{P_L}{P_{max}} = \frac{1-|r_i|^2}{|1-S_{11}r_i|^2} \frac{1-|r_e|^2}{|1-S_{22}r_e|^2} \cdot |S_{21}|^2$$

$$\boxed{g_{TU\,max} = \frac{|S_{21}|^2}{(1-|S_{11}|^2)(1-|S_{22}|^2)}} \quad \text{for } r_i = S_{11}^*, \; r_e = S_{22}^*$$

$$\hookrightarrow g_{TU} = \underbrace{\frac{(1-|r_i|^2)(1-|S_{11}|^2)}{|1-S_{11}r_i|^2}}_{p_i} \, g_{TU\,max} \, \underbrace{\frac{(1-|r_e|^2)(1-|S_{22}|^2)}{|1-S_{22}r_e|^2}}_{p_e}$$

$$\boxed{g_{TU} = p_i \cdot g_{TU\,max} \, p_e} \quad \text{mismatch}$$

$$\rho_i = \frac{z_i - z_{S11}^*}{z_i + z_{S11}} \qquad \rho_e = \frac{z_e - z_{S22}^*}{z_e + z_{S22}}$$

$$\boxed{p = 1-|\rho|^2 = \frac{P}{P_{max}} \qquad |\rho| = \sqrt{1-p}}$$

Circles of constant mismatch:

$$\sigma_i = \frac{|S_{11}|(1-|\rho_i|^2)}{1-|S_{11}\rho_i|^2}$$

$$r_i = \frac{|\rho_i|(1-|S_{11}|^2)}{1-|S_{11}\rho_i|^2}$$

Error estimation for unilateral calculation:

$$s = \frac{|S_{12}||S_{21}||S_{11}||S_{22}|}{(1-|S_{11}|^2)(1-|S_{22}|^2)}$$

$$\downarrow \frac{1}{(1+s)^2} < \frac{g_T}{g_{TU}} < \frac{1}{(1-s)^2}$$